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PARAMETER VARIATION ON MHD OSCILLATORY FLUID FLOW IN A POROUS PARALLEL CHANNEL WITH HEAT

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ABSTRACT

The parameter variation on MHD oscillatory fluid flow in a porous parallel channel with heat was investigated. The governing equations were formulated, and the coupled partial differential equations were converted to ordinary differential equations by adopting perturbation parameters involving the oscillatory frequency terms such as $u = u_o e^{i\omega t}$ and $\theta = \theta_o e^{i\omega t}$ in the channel of flow for velocity and temperature profiles respectively. Numerical simulation was carried out using Mathematica 12 to study the velocity and temperature profiles with some pertinent parameters such as Gr, Pr, M, K, Rd, R. In addition, it is seen that some of the aforementioned parameters influenced the flow profiles in increasing and decreasing fashion which is very important in annexing the usefulness of the parameters to study flow in a porous parallel channels.

Keywords: Mhd, oscillatory, fluid, parameter, porous, heat.

INTRODUCTION

The variation of some physical parameters in scientific detailing of a circumstance is of foremost significance in many fields, for example, building, natural and other mechanical segments. For example, the occasional blood stream in the cardiovascular framework can be portrayed by the recurrence segments of the weight and stream rate beats, and numerous vascular ailments are related with unsettling influences of the neighborhood stream conditions in the veins. A few amazing examinations have been introduced concerning oscillatory stream in a permeable equal channel with heat Bunonyo *et al*. (2017, 2019). Ahmed *et al*. (2015) did an examination to read the numerical investigation for MHD emanating heat/mass vehicle in a Darcian permeable system limited by a swaying vertical surface. Balamurugan *et al*. (2015) detailed a temperamental MHD free convective stream past a moving vertical plate with time subordinate pull and compound response in a slip stream system. Chand *et al*. (2013) have acquired the explanatory answers for oscillatory free convective progression of gooey liquid through permeable medium in a pivoting vertical channel. Cogley *et al*. (1968) considered the differential estimation for radiative warmth move in a non-straight condition dim gas close to harmony. El-Hakien and Hamza (2000, 2011, 2014) examined a MHD oscillatory stream on free convection radiation through a permeable medium. Ibrahim *et al*. (2008) examined the impact of the concoction response and radiation retention on the flimsy MHD free convection stream past a semi–limitless vertical porous moving plate with heat source and attractions.

The warmth and mass exchange impacts on MHD oscillatory stream in a channel loaded up with permeable medium examined by Ibrahim and Makinde (2005, 2015). Kataria and Patel (2016) have researched the impacts of radiation and synthetic response on MHD Casson liquid stream past a swaying vertical plate installed in permeable medium. Malapati and Polarapu (2015) contemplated the time-subordinate MHD free convective warmth and mass exchange in a limit layer stream past a vertical porous plate with warm radiation and substance response. The liquid property impacts gooey dissemination since variety in liquid consistency because of temperature may influence the stream attributes. Gooey dispersal assumes a significant job in different territories, for example, food handling, polymer fabricating, topographical procedures in liquids contained in different bodies, and numerous others. Mansour *et al*. (2008) looks at the impacts of compound response and thick dispersal on MHD common convection streams filled in permeable media. The authors (Misra and Adhikary, 2016; Mohammed *et al*., 2015; Muthucumaraswamy and Manivannan, 2007; Sekhar and Reddy, 2012) looked into a MHD oscillatory channel stream, warmth and mass exchange in a direct in nearness of concoction response and warmth motion.

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Warmth and mass exchange impacts on MHD free convective course through permeable medium within the sight of radiation and thick dispersal was researched by Prasad and Salawu (2007, 2016). Sharma *et al.* (2014) dissected radiative and free convective consequences for MHD move through a permeable medium with intermittent wall temperature and heat generation or ingestion.

MATHEMATICAL FORMULATION

We investigate the parameter variation of MHD oscillator fluid flow in a porous parallel channel with heat with the consideration of the fluid as an incompressible, optically thin radiating and radiation absorbing fluid in an infinite horizontal parallel channel. The x^* - axis is taking along the horizontal channel in the horizontal direction, the y^* - axis is perpendicular to the wall of the channel and the transverse magnetic field of uniformly applied with the strength B_o in the direction parallel to the y^* - axis. Also, we assume that the flow is time dependent and the flow started at zero, and we present the following formulated governing equation:

$$\frac{\partial u^*}{\partial t^*} = \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2}{\rho} u^* - \frac{\upsilon}{k} u^* + g\beta_T (T^* - T_\infty^*) \tag{1}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k_T}{cp} \frac{\partial^2 T^*}{\partial y^{*2}} - Q_o \left(T^* - T_\infty^*\right) - \frac{1}{\rho Cp} \frac{\partial q}{\partial y^*} \tag{2}$$

where $\frac{\partial q_r}{\partial y^*} = 4\alpha^2 (T^* - T_\infty^*)$

The corresponding boundary conditions are:

$$u^* = 0 \ , \qquad T^* = T_w^* \qquad \text{at } y^* = 0 \tag{3}$$

$$u^* = 0 \ , \qquad T^* = T_w^* + \varepsilon \left(T_w^* - T_\infty^*\right) e^{i\omega^* t^*}$$

$$\text{at } y^* = R_0 \tag{4}$$

Using the following non-dimensional parameters:

$$\left.\begin{aligned} & y = \frac{y^*}{R_o}, t = \frac{t^* \upsilon}{R_o^2}, \omega = \frac{R_0^2 \omega^*}{\upsilon}, u = \frac{u^*}{U_o}, \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, M^2 = \frac{\sigma B_o^2 R_o^2}{\rho \upsilon} \\ & Gr = \frac{g\beta_T \left(T_w^* - T_\infty^*\right)}{\upsilon U_0}, R^2 = \frac{4\alpha^2 R_o^2}{k_T}, Pr = \frac{\rho \upsilon c_p}{k_T}, Rd = \frac{Q_o R_o^2}{k_T}, K^2 = \frac{R_o^2}{k} \end{aligned}\right\} \tag{5}$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \left(K^2 + M^2\right) u + Gr\theta \tag{6}$$

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - \left(R^2 + Rd\right) \theta \tag{7}$$

The corresponding boundary conditions are:

$$u = 0 \ , \qquad \theta = 1 + \varepsilon e^{i\omega t} \qquad \text{at } y = 1 \tag{8}$$

$$u = 0 \ , \qquad \theta = 1 \qquad \text{at } y = 0 \tag{9}$$

MATERIALS AND METHODS

Method of Solutions

For us to solve equation (6) and (7), the coupled, non-linear differential equations, we assume an oscillatory behavior inside the channel and as such we consider the following oscillatory condition, as follows:

$$u = u_o e^{i\omega t} \ , \ \theta = \theta_o e^{i\omega t} \tag{10}$$

Substituting equation (10) into equations (6) and (7), we have the following:

$$\frac{\partial^2 u_o}{\partial y^2} - \chi_1^2 u_o = -Gr\theta_o \tag{11}$$

$$\frac{\partial^2 \theta_o}{\partial y^2} - \chi_2^2 \theta_o = 0 \tag{12}$$

where $\chi_1^2 = \left(\left(K^2 + M^2\right) + i\omega\right)$ and

$$\chi_2^2 = \left(\left(R^2 + Rd\right) + Pri\omega\right)$$

The corresponding boundary conditions are:

$$u_o = 0 \ , \quad \theta_o = e^{-i\omega t} \qquad \text{at } y = 0 \tag{13}$$

$$u_o = 0 \ , \quad \theta_o = \varepsilon + e^{-i\omega t} = \phi_1(\varepsilon, \omega) \quad \text{at } y = 1 \tag{14}$$

Solving equations (11) and (12) using the boundary conditions in equations (13) and (14), we have the following functions for velocity and temperature profiles as

$$\frac{\partial^2 u_o}{\partial y^2} - \chi_1^2 u_o = -GrB_1 cosh(\chi_2 y) - GrB_2 sinh(\chi_2 y) \tag{15}$$

$$\theta_o(y) = B_1 \cos h(\chi_2 y) + B_2 \sin h(\chi_2 y) \tag{16}$$

$$B_2 = \frac{(\varepsilon + e^{-i\omega t}) - e^{-i\omega t} \cos h(\chi_2)}{sinh(\chi_2)} = \frac{(\varepsilon + e^{-i\omega t}) - e^{-i\omega t} \cos h(\chi_2)}{sinh(\chi_2)}, B_1 = e^{-i\omega t} \tag{17}$$

Solving equation (15), we have the following homogenous and in homogenous solutions as:

$$u_{oh}(y) = B_3 cosh(\chi_1 y) + B_4 sinh(\chi_1 y) \tag{18}$$

$$u_{op}(y) = \left[\frac{GrB_1}{\left(\chi_2^2 - \chi_1^2\right)}\right] cosh(\chi_2 y) + \left[\frac{GrB_2}{\left(\chi_2^2 - \chi_1^2\right)}\right] sinh(\chi_2 y) \tag{19}$$

So that the general solution for equation (15) is as follows:

$$u_o(y) = B_3 cosh(\chi_1 y) + B_4 sinh(\chi_1 y) + \left[\frac{GrB_1}{\left(\chi_1^2 - \chi_2^2\right)}\right] cosh(\chi_2 y) + \left[\frac{GrB_2}{\left(\chi_1^2 - \chi_2^2\right)}\right] sinh(\chi_2 y) \tag{20}$$

where $B_5 = \frac{GrB_1}{\left(\chi_1^2 - \chi_2^2\right)}, B_6 = \frac{GrB_2}{\left(\chi_1^2 - \chi_2^2\right)}$, $B_3 = \left(\frac{GrB_1}{\left(\chi_2^2 - \chi_1^2\right)}\right)$

$$B_4 = \left(\frac{Gr}{\left(\chi_2^2 - \chi_1^2\right) sinh(\chi_1)}\right) \left(B_1 \left(cosh(\chi_2) - cosh(\chi_1)\right) + B_2 sinh(\chi_2)\right)$$

RESULTS

The reduced nonlinear differential equations governing the fluid flow were solved analytically using perturbation technique. The simulation was carried out using the software called Mathematica, version 12 and for different values of pertinent physical parameters such as Rd, M, Gr, K, R and ω. The influence of the resulting parameters on velocity and temperature, skin friction and the rate of heat transfer are investigated.

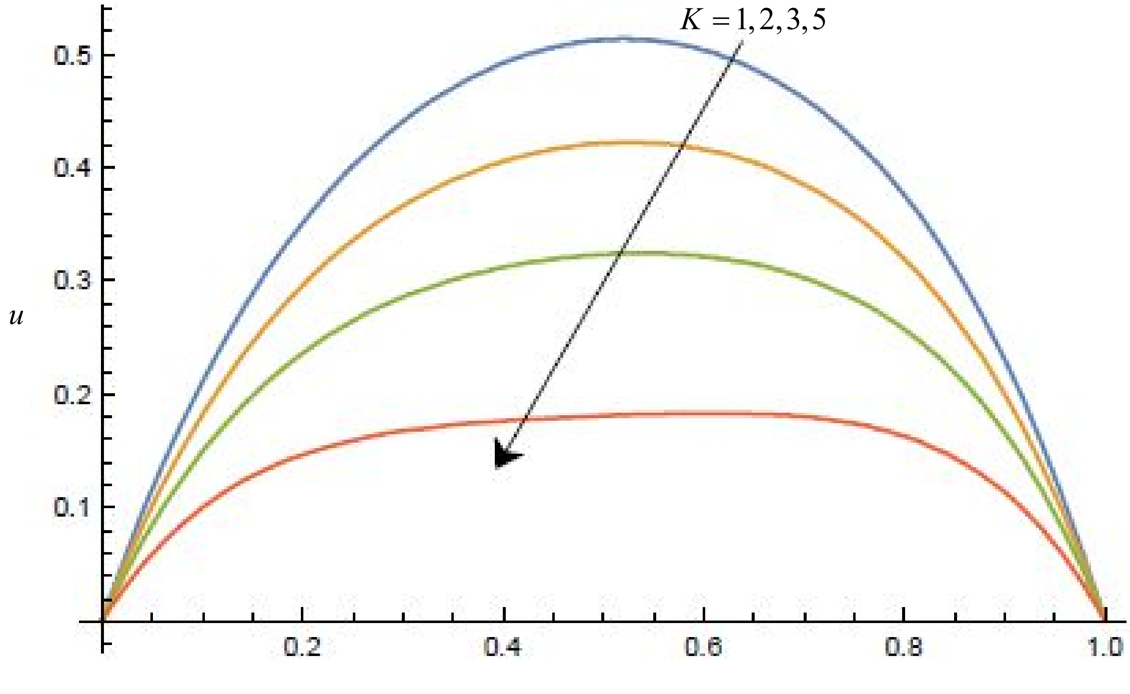

Fig. 1. Influence of K on velocity profile with other valuable parameters.

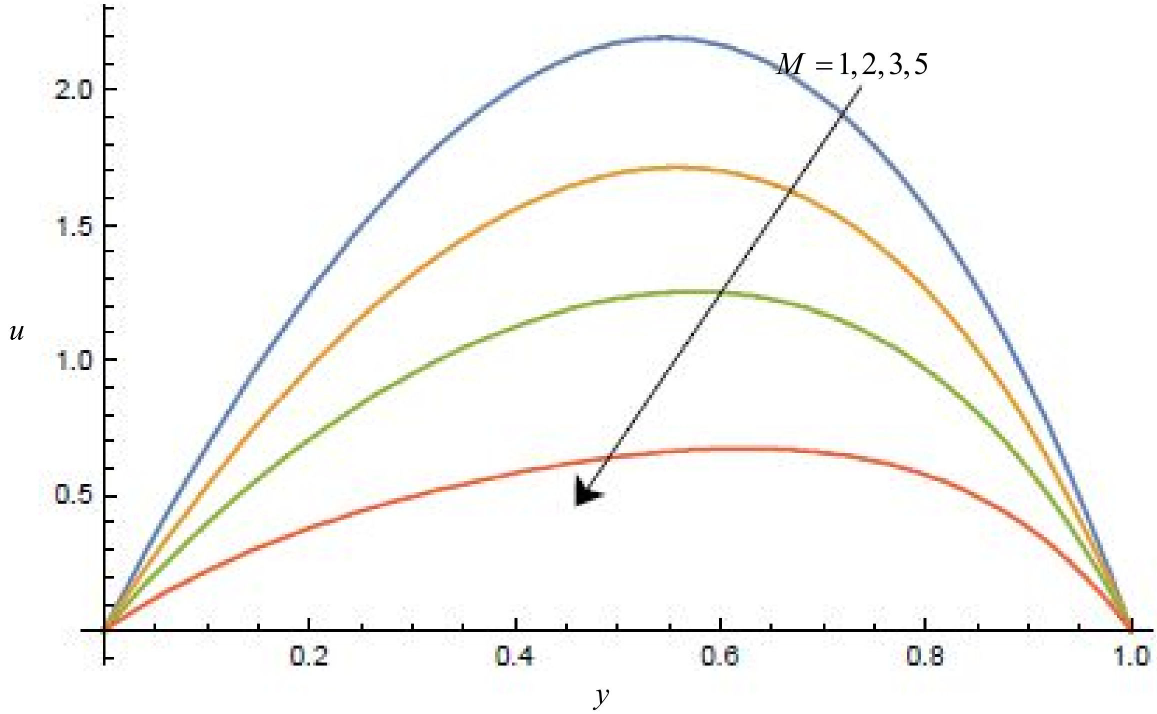

Fig. 2. Influence of M on velocity profile with other valuable parameters.

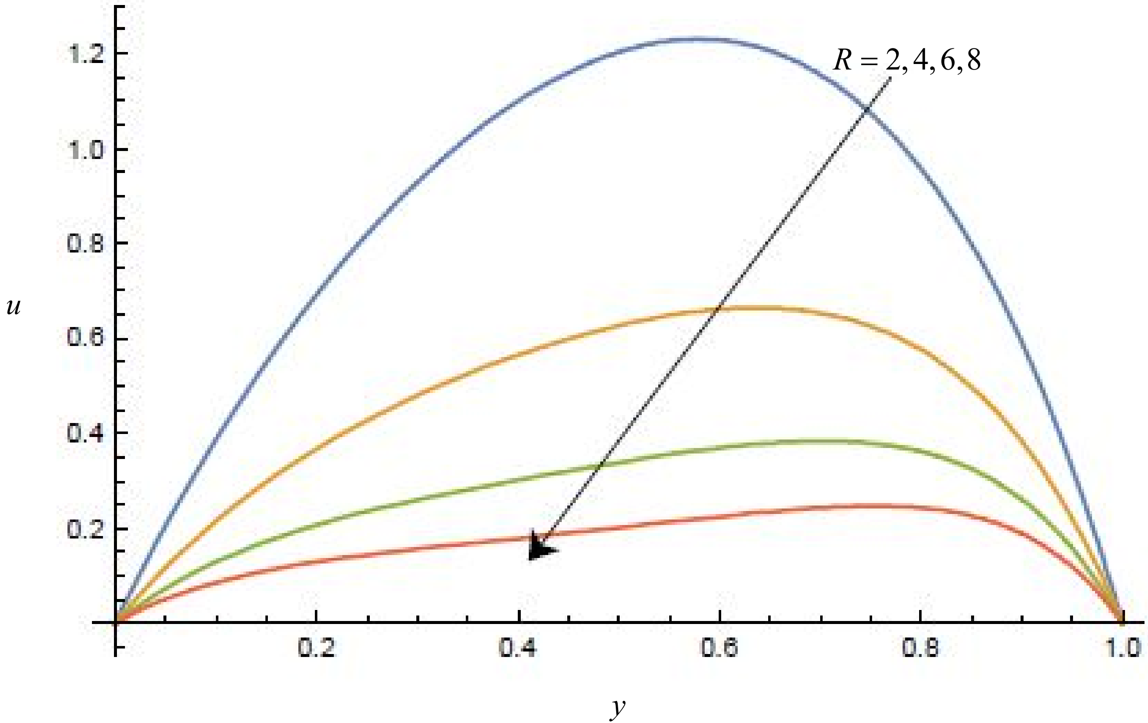

Fig. 3. Influence of R on velocity profile with other valuable parameters.

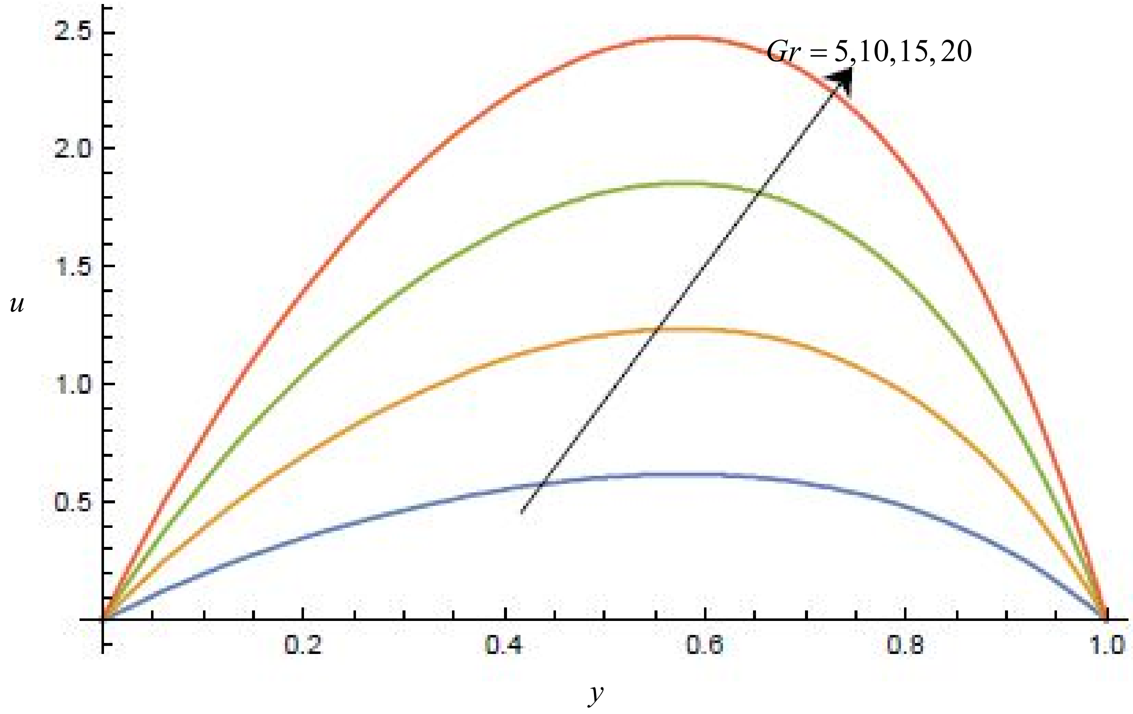

Fig. 4. Influence of Gr on velocity profile with other valuable parameters.

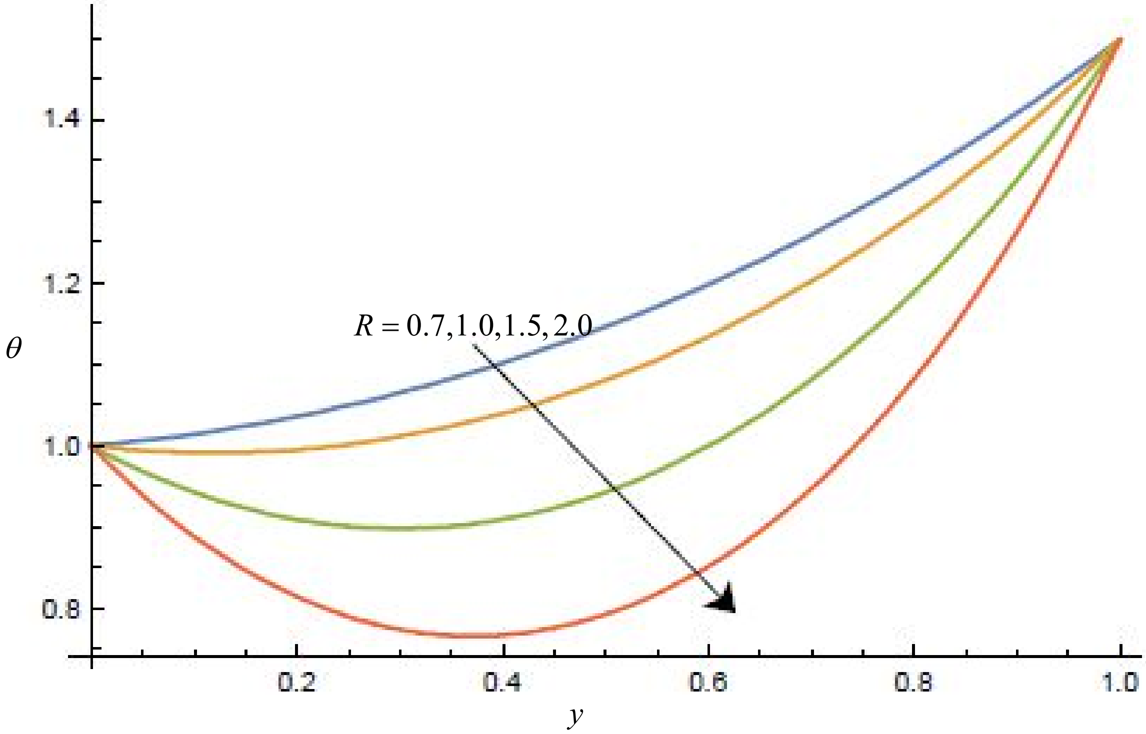

Fig. 5. Influence of R on temperature profile with other parameters.

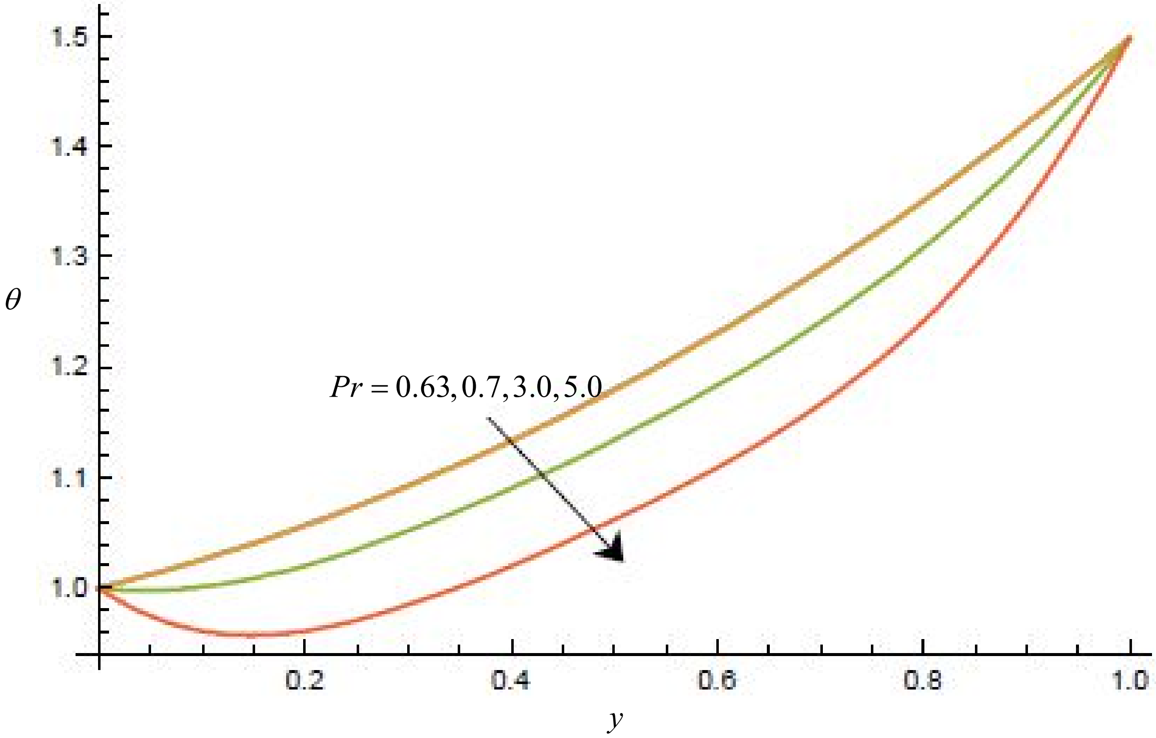

Fig. 6. Influence of Pr on velocity profile with other valuable parameters.

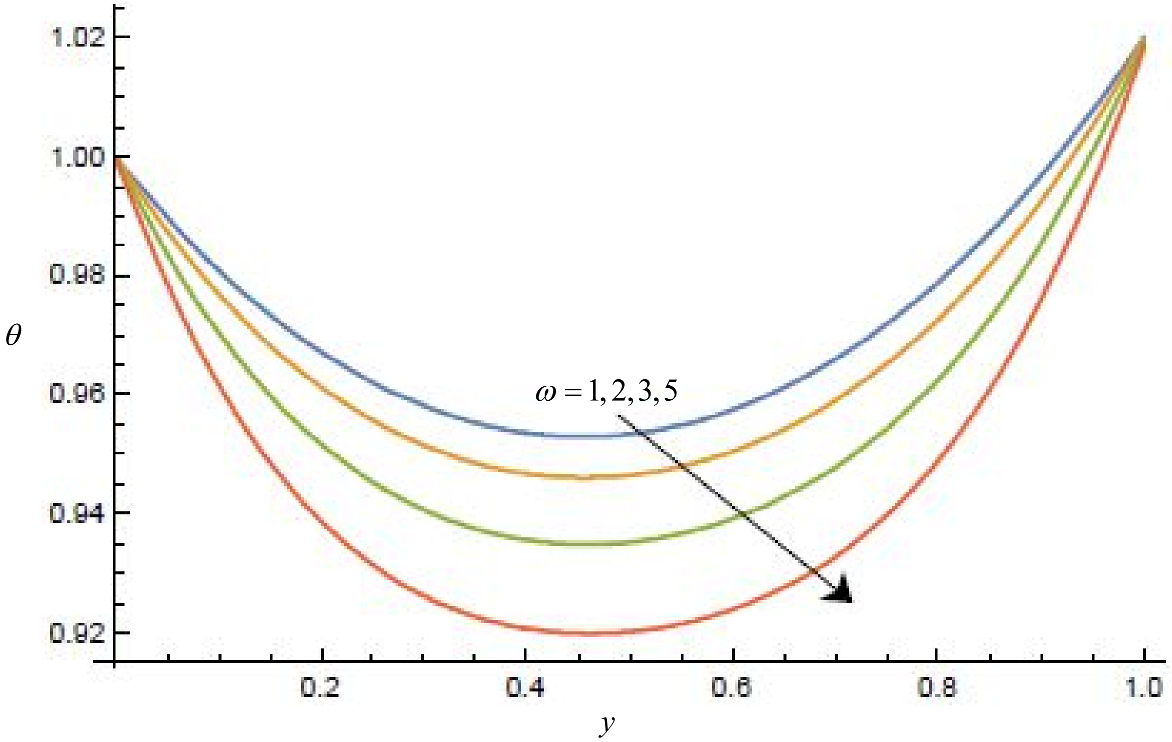

Fig. 7. Influence of ω on velocity profile with other valuable parameters.

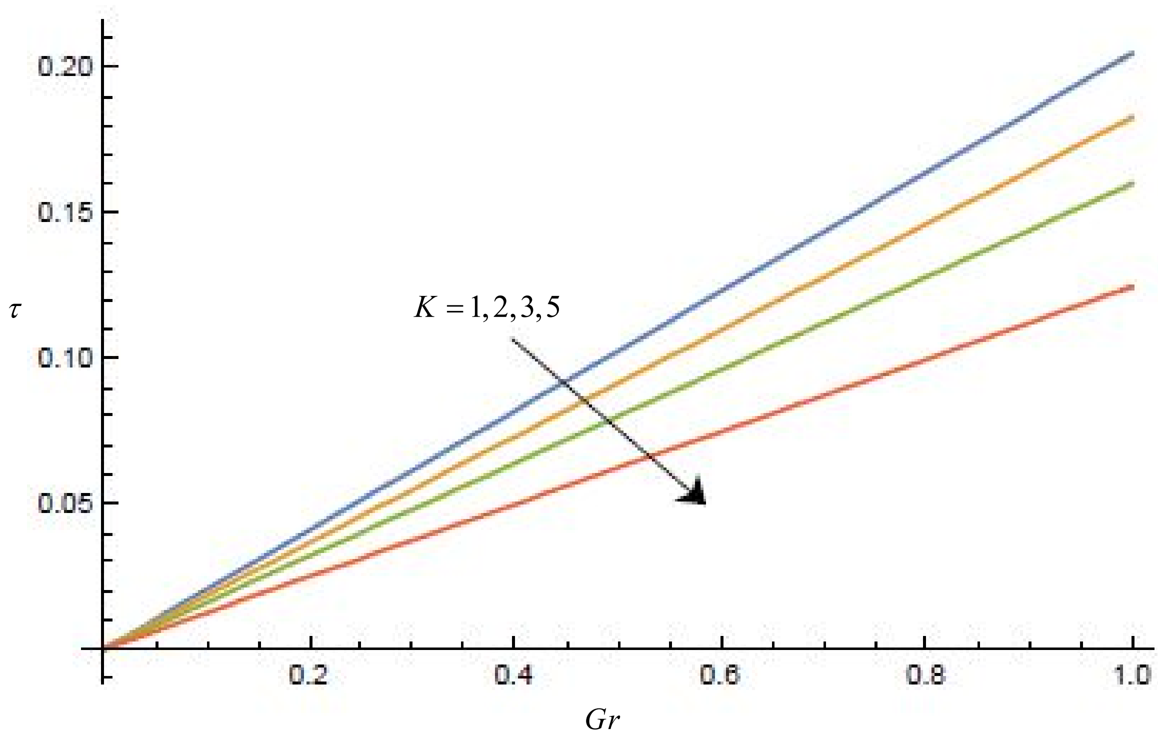

Fig. 8. Influence of K on Shear Stress τ against Grashof number Gr with other parameters.

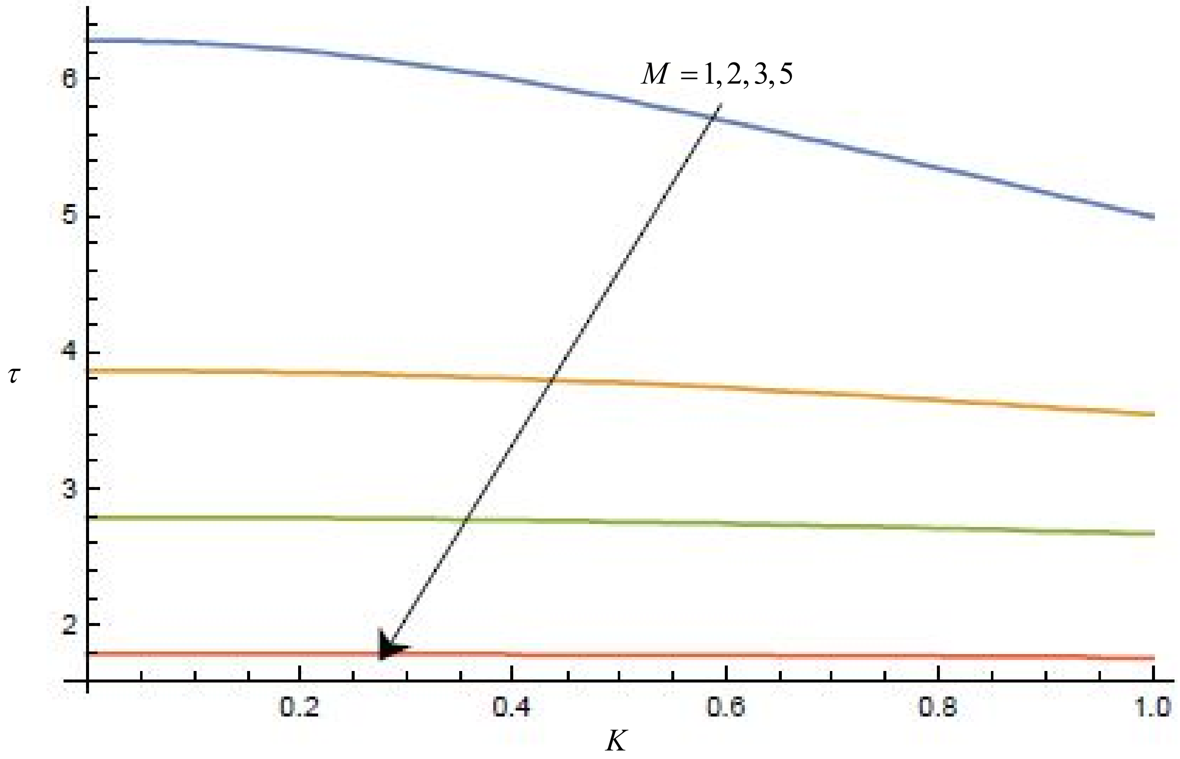

Fig. 9. Influence of M on Shear Stress τ against porosity K with other parameters.

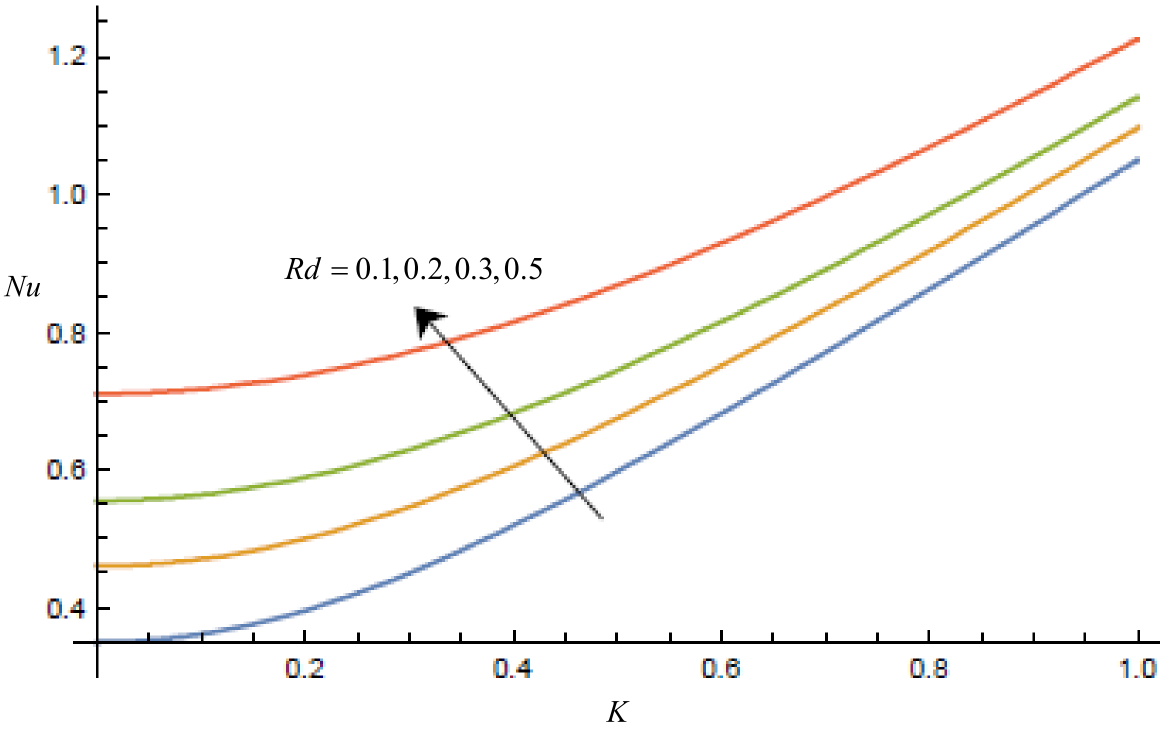

Fig. 10. Influence of Rd on Nu against K with other parameters.

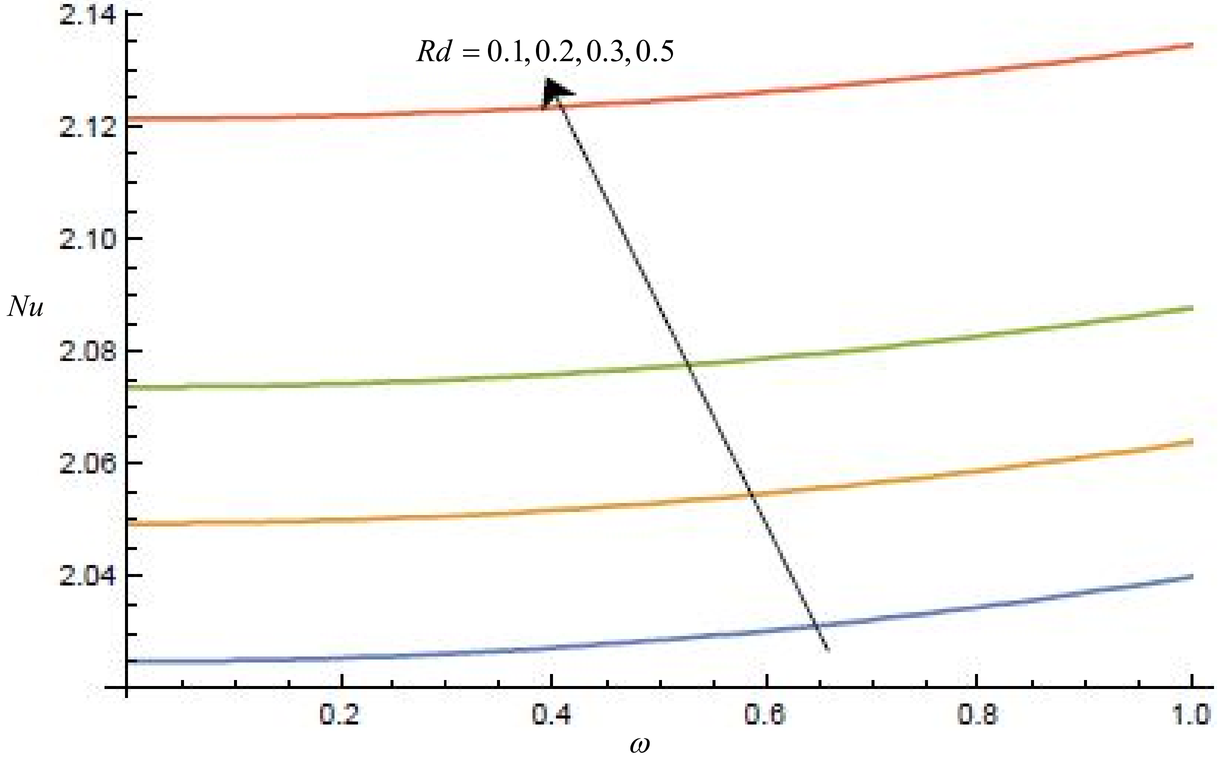

Fig. 11. Influence of Rd on Nu against ω with other parameters.

DISCUSSION

In the preceding section, we have been able to carry out numerical simulation of the analytical solution considering the variation of the pertinent parameters as mentioned, the results are discussed as follows:

In Figure 1, it is observed that the velocity profile insignificantly reduced with the increasing values of the permeability parameter K. It is a clear indication that velocity profile becomes thick for the increasing values of K. Figure 2 also demonstrates the influence of the Hartmann number M on the velocity profile. It is observed that the velocity profile reduces with increasing values of M. It is so because the application of transverse magnetic field plays the role of resistive force or Lorentz force just like the drag force that acts opposite to the direction of fluid motion.

Figure 3 depicts that an increase in radiation absorption parameter R leads to decreasing fluid boundary layer. It is so because large values of R corresponds to an increasing dominance of the conduction over R thereby decreasing the boundary thickness of the momentum boundary layer.

It is also shown in Figure 4 shows the effect of Gr on the velocity profile. It is seen that the velocity profile increased significantly with the increasing values of $Gr > 0$. The fluid velocity increase is due to the enhancement of thermal boundary force. The temperature is so significant in the sense that it can help in enhancing the fluid behavior and as such we take a look at Figure 5, this figure depicts that radiation absorption parameter, R influenced the temperature beginning from $\theta = 1$ and gets to a maximum value as $y = 1$.

Figure 6 clearly indicates that the increase in Pr leads to the corresponding decreasing in the temperature profile. This is so since at high Prandtl number, Pr the thermal difference of the fluid is reduced and causes weak penetration of heat inside the fluid. Figure 7 depicts also that increase in oscillatory parameter led to a corresponding increasing in temperature profiles.

We could see in Figure 8 and Figure 9 it can be clearly seen that permeability parameter and Hartmann number caused a decrease in shear stress against Grashoff and permeability, respectively. Figure 10 and Figure 11 tells that the rate of heat transfer increases against permeability and oscillatory parameters respectively as the radiation heat parameter is increased.

CONCLUSION

The investigation of parameter variation on MHD oscillator fluid flow in a porous parallel channel with heat has been studied. The governing equations are solved analytically and numerical simulations were carried out using mathematical software called Mathematic version 12, to study/observe the effect of the governing parameters on the velocity, temperature profile, the skin friction and the rate of heat transfer are presented graphically.

We can conclude from the results that:

1. The velocity profile was enhanced for the increasing value of $Gr > 0$ while the contrast trend is found on M, K, R and Pr.
2. The temperature profile decreases with the increasing values of R, Pr and ω.

The increasing values of K and M caused the shear stress to reduce. However, the radiation parameter and the radiation absorption parameter values increase caused a corresponding increase in the rate of heat transfer.

It is presumed that, with the help of our formulated model, the physics of the flow through a parallel channel can be utilized as the basis for many scientific applications. The results are of great interest in geophysics and other physical sciences.

NOMENCLATURE

u^*	Dimensional velocity profile
u_o	Perturbed velocity profile
x^*, y^*	Dimensional distances
R	Heat absorption parameter
Rd	Heat absorption constant
k_T	Thermal conductivity of the fluid
K	Porosity parameter
Gr	Thermal Grashof number
up	Wall dimensionless velocity
B_o	Strength of applied magnetic field
Cp	Specific heat capacity at constant pressure
M	Magnetic parameter
T^*	Temperature of the fluid
T_∞^*	Temperature of the fluid far from the plate

Greek Symbols

υ	Kinematic viscosity
Pr	Prandtl number
μ	Dynamic viscosity
g	Acceleration due to gravity
ω	Oscillatory frequency
β_T	Thermal expansion coefficient
θ	Dimensionless temperature
θ_o	Dimensionless perturbed temperature
ρ	Density of the fluid

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